Research on portfolio selection

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1. OLPS

1.1 Introduction

- OLPS is an open-source toolbox for "On-Line Portfolio Selection" presented by Bin Li.
- OLPS is built on matlab.
- OLPS implements a collection of classical and state-of-the-art strategies powered by machine learning algorithms.

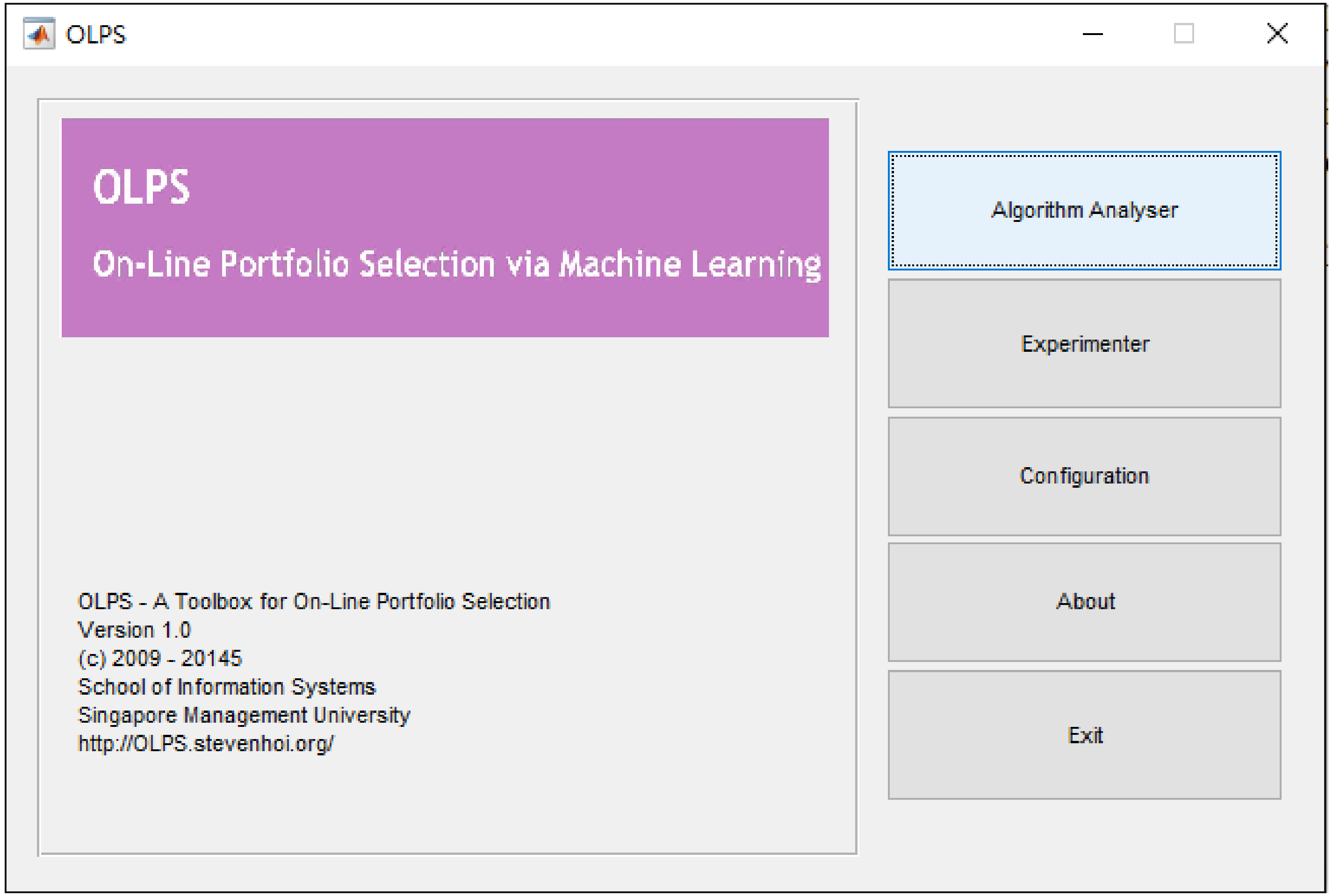
1. OLPS

1.2 Property

- Three types of calling method are provided, including OLPS_gui.m, OLPS_pgui.m and OLPS_cli.m
- Returns : daily returns and cumulative returns
- Risk Analysis
  - Sharpe Ratio, Calmar Ratio, Sortino Ratio, Value at Risk, and Maximum Draw Down.
- Portfolio Analysis

1. OLPS

1.2 Property



Analysis an algorithm

Compare multiple algorithms

2. Portfolio selection

n *periods*, m assets.

$t = 1, \ldots, n$, the asset (close) prices is represented by $\boldsymbol{P}_t \in R_+^m$

Close price $p_{t,i}$ relative price $x_{t,i}$

$$\begin{bmatrix} & \cdots & \\ & \cdots & \\ p_{t,i-1} & p_{t,i} & p_{t,i+1} \\ p_{t+1,i-1} & p_{t+1,i} & p_{t+1,i+1} \\ & \cdots & \\ & \cdots & \end{bmatrix} \quad x_{t,i} = \frac{p_{t,i}}{p_{t-1,i}} \Rightarrow \begin{bmatrix} & \cdots & \\ & \cdots & \\ x_{t,i-1} & x_{t,i} & x_{t,i+1} \\ x_{t+1,i-1} & x_{t+1,i} & x_{t+1,i+1} \\ & \cdots & \\ & \cdots & \end{bmatrix}$$

like this

data ×

1276x25 double

	1	2	3
1	1.0177	0.9943	1.0095
2	0.9867	1.0057	1.0063
3	1.0084	0.9881	0.9969
4	0.9908	1.0072	0.9859
5	0.9756	0.9732	0.9714
6	1.0276	1.0197	1.0344

2. Portfolio selection

For the t^{th} period, portfolio vector is $\boldsymbol{b}_t = (b_{t,1}, \ldots, b_{t,m})$
$\boldsymbol{b}_t \in \Delta_m$, where $\Delta_m = \{\boldsymbol{b}_t : \boldsymbol{b}_t \succcurlyeq 0, \sum_{i=1}^{m} b_{t,i} = 1\}$

Daily return $$s_t = \boldsymbol{b}_t^{\intercal} \boldsymbol{x}_t = \sum_{i=1}^{m} b_{t,i} x_{t,i}$$

Final return $$S_n(b_1^n, x_1^n) = S_0 \prod_{t=1}^{n} \boldsymbol{b}_t^{\intercal} \boldsymbol{x}_t$$

2. Portfolio selection

Protocol 1: On-line portfolio selection.

Input: $\mathbf{x}_1^n$: Historical market price relative sequence
Output: S_n: Final cumulative wealth

Initialize $S_0 = 1, \mathbf{b}_1 = \left(\frac{1}{m}, \dots, \frac{1}{m}\right)$;
for $t = 1, 2, \dots, n$ **do**
    Portfolio manager learns a portfolio $\mathbf{b}_t$;
    Market reveals a price relative vector $\mathbf{x}_t$;
    Portfolio incurs period return $s_t = \mathbf{b}_t^\top \mathbf{x}_t$ and updates cumulative return $S_t = S_{t-1} \times \left(\mathbf{b}_t^\top \mathbf{x}_t\right)$;
    Portfolio manager updates his/her decision rules;
end

3. Strategies

Categories	Strategies	Sections	Strategy Names
Benchmarks	Uniform Buy And Hold	3.1.1	ubah
	Best Stock	3.1.2	best
	Uniform Constant Rebalanced Portfolios	3.1.3	ucrp
	Best Constant Rebalanced Portfolios	3.1.4	bcrp
Follow the Winner	Universal Portfolios	3.2.1	up
	Exponential Gradient	3.2.2	eg
	Online Newton Step	3.2.3	ons
Follow the Loser	Anti Correlation	3.3.1	anticor/anticor_anticor
	Passive Aggressive Mean Reversion	3.3.2	pamr/pamr_1/pamr_2
	Confidence Weighted Mean Reversion	3.3.3	cwmr_var/cwmr_stdev
	On-Line Moving Average Reversion	3.3.4	olmar1/olmar2
Pattern Matching	Nonparametric Kernel-based Log-optimal	3.4.1	bk
	Nonparametric Nearest Neighbor Log-optimal	3.4.2	bnn
	Correlation-driven Nonparametric Learning	3.4.3	corn/cornu/cornk
Others	M0		m0
	T0		t0

3. Strategies

3.1 Benchmarks ·Uniform Buy and Hold

UBH buys the set of assets at the beginning and holds the allocation of assets till the end of trading periods.

$$S_n\left(BAH\left(\mathbf{b}_1\right)\right) = \mathbf{b}_1 \cdot \left(\bigodot_{t=1}^{n} \mathbf{x}_t\right),$$

$$\mathbf{b}_{t+1} = \frac{\mathbf{b}_t \odot \mathbf{x}_t}{\mathbf{b}_t^\top \mathbf{x}_t},$$

3. Strategies

3.2 Benchmarks ·Best Stock

BS is a special BAH strategy that buys the best stock in hindsight.

$$S_n\left(Best\right) = \max_{\mathbf{b}\in\Delta_m} \mathbf{b} \cdot \left(\bigodot_{t=1}^{n} \mathbf{x}_t\right) = S_n\left(BAH\left(\mathbf{b}^{\circ}\right)\right),$$

$$\mathbf{b}^{\circ} = \arg\max_{\mathbf{b}\in\Delta_m} \mathbf{b} \cdot \left(\bigodot_{t=1}^{n} \mathbf{x}_t\right).$$

3. Strategies

3.3 Benchmarks ·Uniform Constant Rebalanced Portfolios

CRP is a fixed proportion strategy, which rebalances to a preset portfolio at the beginning of every period.

$$\mathbf{b}_1^n = \{\mathbf{b}, \mathbf{b}, \ldots\}$$

$$S_n\left(CRP\left(\mathbf{b}\right)\right) = \prod_{t=1}^{n} \mathbf{b}^\top \mathbf{x}_t.$$

(UCRP) chooses a uniform portfolio as the preset portfolio

$$\mathbf{b} = \left(\frac{1}{m}, \ldots, \frac{1}{m}\right).$$

3. Strategies

3.3 Benchmarks ·Uniform Constant Rebalanced Portfolios

Table 1 Motivating example of CRP to show the mean reversion trading idea

# Day	Relative (A,B)	BCRP	BCRP return	Wealth proportion	Notes
1	$(\frac{1}{2}, 2)$	$(\frac{1}{2}, \frac{1}{2})$	$\frac{5}{4}$	$(\frac{1}{5}, \frac{4}{5})$	$B \longrightarrow A$
2	$(2, \frac{1}{2})$	$(\frac{1}{2}, \frac{1}{2})$	$\frac{5}{4}$	$(\frac{4}{5}, \frac{1}{5})$	$A \longrightarrow B$
3	$(\frac{1}{2}, 2)$	$(\frac{1}{2}, \frac{1}{2})$	$\frac{5}{4}$	$(\frac{1}{5}, \frac{4}{5})$	$B \longrightarrow A$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

3. Strategies

3.4 Benchmarks ·Best Constant Rebalanced Portfolios

BCRP sets the portfolio as the portfolio that maximizes the terminal wealth in hindsight.

$$\mathbf{b}_1^n = \{\mathbf{b}, \mathbf{b}, \dots\}$$

$$\mathbf{b}^\star = \arg\max_{\mathbf{b}^n \in \Delta_m} \log S_n\left(CRP\left(\mathbf{b}\right)\right) = \arg\max_{\mathbf{b} \in \Delta_m} \sum_{t=1}^{n} \log\left(\mathbf{b}^\top \mathbf{x}_t\right).$$

$$S_n\left(BCRP\right) = \max_{\mathbf{b} \in \Delta_m} S_n\left(CRP\left(\mathbf{b}\right)\right) = S_n\left(CRP\left(\mathbf{b}^\star\right)\right),$$

3. Strategies

3.5 Follow the Winner ·Universal Portfolios

UP uniformly buys and holds the whole set of CRP experts within the simplex domain.

In some sense, Cover's algorithm divides its money evenly among all CRPs

$$\hat{\mathbf{b}}_1 = \left(\frac{1}{m}, \frac{1}{m}, \ldots, \frac{1}{m}\right), \qquad \hat{\mathbf{b}}_{k+1} = \frac{\int \mathbf{b} S_k(\mathbf{b})\, d\mathbf{b}}{\int S_k(\mathbf{b})\, d\mathbf{b}},$$

3. Strategies

3.6 Follow the Winner ·Exponential Gradient

EG tracks the best stock and adopts regularization term to constrain the deviation from previous portfolio.

$$\mathbf{b}_{t+1} = \arg\max_{\mathbf{b}\in\Delta_m} \quad \eta \log \mathbf{b}\cdot\mathbf{x}_t - R\left(\mathbf{b}, \mathbf{b}_t\right),$$

$$R\left(\mathbf{b}, \mathbf{b}_t\right) = \sum_{i=1}^{m} b_i \log \frac{b_i}{b_{t,i}}.$$

$$b_{t+1,i} = b_{t,i} \exp\left(\eta \frac{x_{t,i}}{\mathbf{b}_t \cdot \mathbf{x}_t}\right)/Z, \quad i = 1, \ldots, m,$$

3. Strategies

3.7 Follow the Winner ·Online Newton Step

ONS tracks the best CRP to date and adopts a L2-norm regularization to constrain portfolio's variability

$$\mathbf{b}_{t+1} = \arg\max_{\mathbf{b}\in\Delta_m} \sum_{\tau=1}^{t} \log\left(\mathbf{b}\cdot\mathbf{x}_\tau\right) - \frac{\beta}{2}\left\|\mathbf{b}\right\|.$$

$$\mathbf{b}_1 = \left(\frac{1}{m},\ldots,\frac{1}{m}\right), \quad \mathbf{b}_{t+1} = \Pi_{\Delta_m}^{\mathbf{A}_t}\left(\delta\mathbf{A}_t^{-1}\mathbf{p}_t\right),$$

$$\mathbf{A}_t = \sum_{\tau=1}^{t}\left(\frac{\mathbf{x}_\tau\mathbf{x}_\tau^\top}{\left(\mathbf{b}_\tau\cdot\mathbf{x}_\tau\right)^2}\right) + \mathbf{I}_m, \quad \mathbf{p}_t = \left(1+\frac{1}{\beta}\right)\sum_{\tau=1}^{t}\frac{\mathbf{x}_\tau}{\mathbf{b}_\tau\cdot\mathbf{x}_\tau},$$

3. Strategies

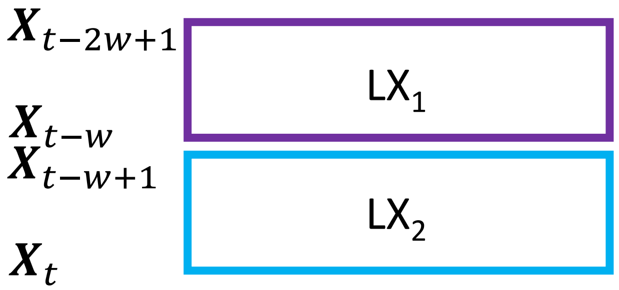
3.8 Follow the Loser ·Anti correlation

$$\mathsf{LX}_1 = \log(\mathbf{x}_{t-2w+1}), \ldots, \log(\mathbf{x}_{t-w})^T \text{ and } \mathsf{LX}_2 = \log(\mathbf{x}_{t-w+1}), \ldots, \log(\mathbf{x}_t)^T,$$

cross-correlation matrix

$$M_{cov}(i,j) = (\mathsf{LX}_1(i) - \mu_1(i))^T(\mathsf{LX}_2(j) - \mu_2(j));$$

$$M_{cor}(i,j) \begin{cases} \frac{M_{cov}(i,j)}{\sigma_1(i)\sigma_2(j)} & \sigma_1(i), \sigma_2(j) \neq 0; \\ 0 & \text{otherwise.} \end{cases}$$



3. Strategies

3.8 Follow the Loser ·Anti correlation

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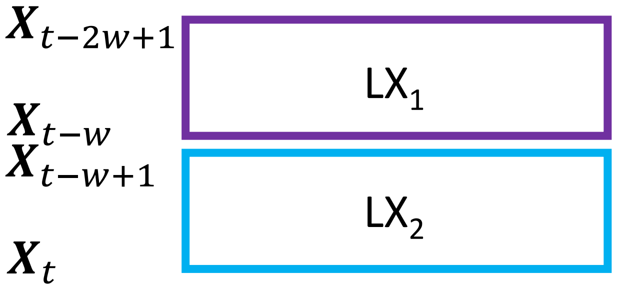


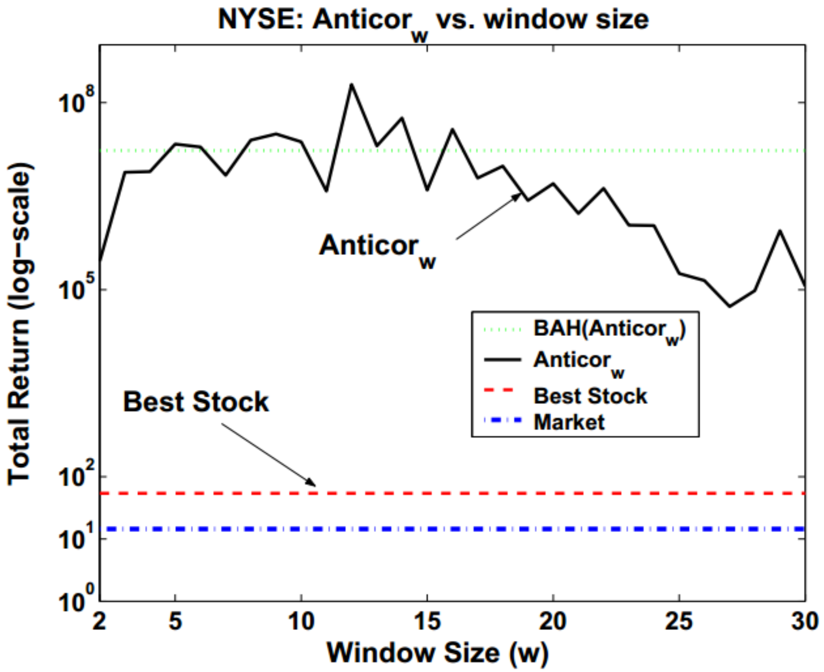
$$\mathsf{claim}_{i\to j} = M_{cor}(i,j) + A(i) + A(j)$$ A(h) = |Mcor(h, h)| if Mcor(h, h) < 0, else 0.

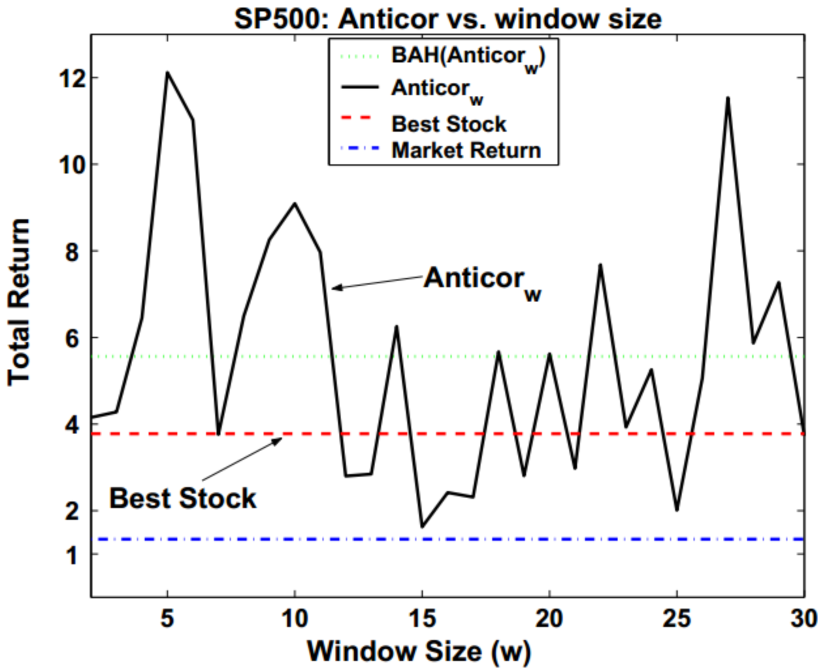
$$\mathbf{b}_{t+1}(i) = \tilde{\mathbf{b}}_t(i) + \sum_{j\neq i}[\mathsf{transfer}_{j\to i} - \mathsf{transfer}_{i\to j}]$$

$$\mathsf{transfer}_{i\to j} = \tilde{\mathbf{b}}_t(i) \cdot \mathsf{claim}_{i\to j} / \sum_j \mathsf{claim}_{i\to j}$$

3. Strategies

3.8 Follow the Loser ·Anti correlation





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Thank you for your attention！

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